21 November
Monday

Chapter
Waves

1) Wave motions: It's a form of disturbance which travels through a mediam due to repeated periodic motion of the particles ~~of~~ their

In a ~~way~~ wave motion, energy and momentum will be transferred from one point to another without actual transportation of matter.

2) Types of Wave motion:

a) Mechanical Wave: Waves which require a ~~to~~ a material medium for its propogation is called mechanical wave.
eg:- Sound, water waves.

b) Electromagnetic waves: Waves which doesn't require any material medium for its propogation is called electromagnetic waves.
eg:- Radio, X-Ray

c) Matter waves: Waves which is associated with moving particles of matter such as electrons protons and nuetrons is called matter wave.

3) Types of mechanical waves:

a) Transverce wave: A transverse wave is a wave in which indiviual particles of median viberates in a direction perpendicular to the direction of propogation of wave. A transverse wave travel through crust and ~~trump~~ trough.

b) Longitiudnal Wave: It is a wave in which the indiuival particles viberates along the direction of propagation of a wave. A Longituadinal wave travels through a median in form of compression and rare fractions.

4) Characteristics of Wave motion:

a) ~~W~~ Amplitude: It is the maximum displacement from mean position.
SI unit is m.

b) Wavelength (λ): It's the distance between two concecutive crust or ~~trahf~~ trough.
SI unit: meter (m)

c) Frequency (ν): Number of viberations per second.
SI unit: Hz (Hertz)

5) Displacement relation for a wave.

$$Y = A \sin(\omega t - Kx)$$

K = propagation constant = $\frac{2\pi}{\lambda}$
A = Amlitude
$\omega = 2\pi\nu$
T = time period

~~It~~ $K = \frac{2\pi}{\lambda}$

6) A wave travelling along a string is described by $Y = 0.005 \sin(80x - 3t)$, calculate the amplitude, Wavelength, time period, frequency and the Direction in which wave is.

Sol. $A = 0.005m$

$\omega = -3$

$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{3} = 20.92$, $\left[\omega = \frac{2\pi}{T}\right]$

$\omega = 2\pi \nu$

$-3 = 2 \times \frac{22}{7} \nu$

$-3 = 3.14 \times 2\nu$

$-3 = 6.28\nu$

$\frac{-3}{6.28} = \nu$

$\nu = -0.47 Hz$

Wave is travelling reverse direction.

7) Superposition principal: It states that when two or more waves travelling through a medium superimpose one another, a new wave is formed in which resultant displacement $(\vec{Y})$ at any instant is equal to the vector sum of displacements $(\vec{Y_1}, \vec{Y_2})$ due to individual waves at that instant.

That is, $\vec{Y} = \vec{Y_1} + \vec{Y_2}$

*When crust of a wave falls on crust

of other, the resultant amplitude of wave is $a_1 + a_2$.

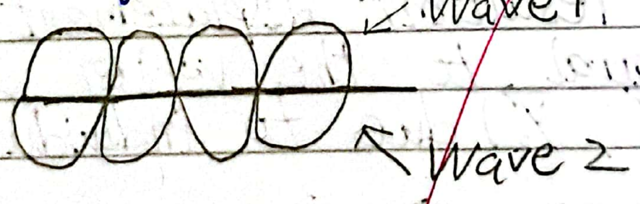
When crust of one wave falls through the other, the resutant amplitude of wave is $a_1 - a_2$.

8) <u>Application of super position principal</u>:

a. Interference
b. Standing Waves
c. Beats

a. <u>Interference</u>: It is the phenomena of re-distribution of energy in a medium due to superposition of waves. At the points where resultant intensity is max. Interference is said to be constructive at the points where resultance intensity is min. The interference is said to be destructive. [deleted portion]

b. <u>Standing waves</u>: When two travelling waves or progressive waves of same amplitude and wavelength travelling with same speed in opposite direction along a same straight line superimpose, they give rise to a new wave. The wave formed is called standing waves or stationary waves.



i. Characteristics of Standing Waves.

→ There is no transfer of energy along the wave in the medium. The waves don't propagate in any medium.

→ Nodes are points in the medium in standing waves that are at rest permantly.

→ The distance between two consective notes is $\lambda/2$.

→ Antinodes are points in the medium that have maximum amplitude of vibration. The distance between two con secutive antinodes is $\lambda/2$.

ii. Equation of a standing wave.

$$y_1 = A \sin(Kx - \omega t) \quad [K = \text{Propagation constant}]$$

$$y_2 = A \sin(Kx + \omega t) \quad \text{(Wave is travelling in opposite direction)}$$

$$\vec{y} = \vec{y_1} + \vec{y_2}$$

$$= A \sin(Kx - \omega t) + A(\sin Kx + \omega t)$$

$$= A\left[2 \sin \frac{Kx - \omega t + Kx + \omega t}{2} \times \cos \frac{Kx - \omega t - Kx - \omega t}{2}\right]$$

$$\left(\sin A + \sin B = 2 \sin \frac{(A+B)}{2} \cos \frac{(A-B)}{2}\right)$$

$$y = 2A \sin \frac{2Kx}{2} \cos \frac{-2\omega t}{2}$$

$$\vec{Y} = 2A \sin Kx \cos \omega t$$

[Only equation is required.]

$Amp = 2A\sin Kx$

iii. Derive an equation for standing waves when a stretched string is fixed in both sides

$y = 2A\sin Kx \cos\omega t$

if $x = 0$

$y = 2A \sin Kx \times 0 \cos\omega t$

$y = 2A \sin 0 \cos\omega t = 0 \times \cos\omega t$

$y = 0$, nodes will be formed.

Putting $y = 0$ in $y = 2A\sin Kx$

$0 = 2A\sin Kx$

$\sin n\pi = 2A\sin Kx$

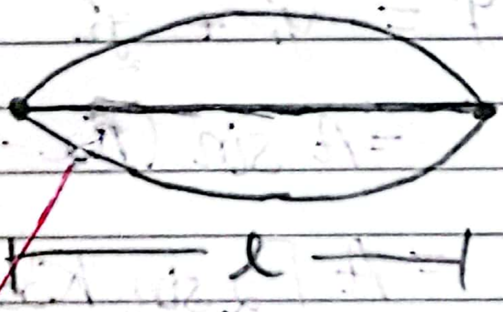
$n\pi = \frac{2\pi x}{\lambda}$ $\quad \left[K = \frac{2\pi}{\lambda}\right]$

$\lambda = \frac{2l}{n}$

First mode of vibrations

$\lambda = \frac{2l}{n}$

Taking $n = 1$



$\lambda = 2l$

$\Rightarrow l = \frac{\lambda}{2}$

From the diagram,

$l = \frac{\lambda}{2}$

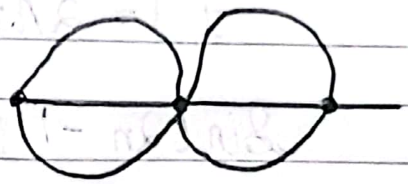
$v_1 = \frac{V}{\lambda} = \frac{V}{2l}$

$v_1 = \frac{V}{2l}$, First Harmonic Frequency.

Second mode of vibration

$\lambda_2 = \frac{2l}{2} = l$ $\because$ when $n = 2$

$\lambda_2 = l$



$v_2 = \frac{V}{\lambda_2}$ $v = 2v_1$

$v_2 = \frac{V}{l}$

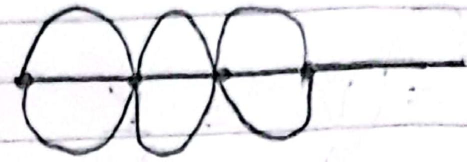
Second Harmonic motion
(aka first overtone)

Third mode of vibrations.

$\lambda = \frac{2l}{n}$, when $n=3$

$\lambda = \frac{2l}{3}$, $l = \frac{3\lambda}{2}$

$$v_3 = \frac{V}{\lambda} = \frac{V}{\frac{2l}{3}} = \frac{3V}{2l}$$



$$v_3 = 3v_1$$

Third Harmonic motion

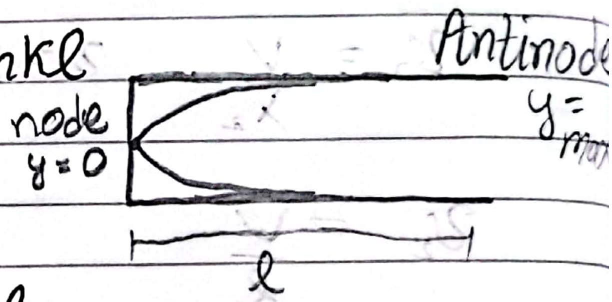
iv. Standing wave - Derive an equation when wave are formed in closed organ pipe.

Sol. $y = 2A \sin K \times 0 \cos \omega t = 0$

$$\sin 0 = 2A \sin Kl$$

$$\pm 1 = 2A \sin Kl$$

$$\sin (2n-1)\frac{\pi}{2} = 2A \sin Kl$$



$$(2n-1)\frac{\pi}{2} = Kl$$

$$(2n-1)\frac{\pi}{2} = \frac{2\pi}{\lambda} \times l$$

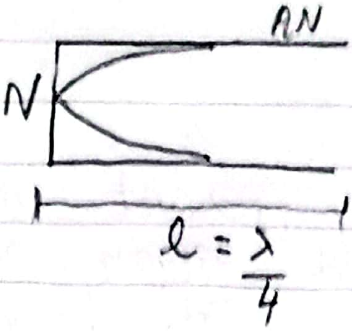
$$(2n-1)\lambda = 4l$$

$$\lambda = \frac{4l}{2n-1}$$

First mode of Vibration

$$n=1, \quad \lambda_1 = \frac{4l}{2\times1-1} = 4l$$

$$l = \frac{\lambda}{4}$$



$$\upsilon_1 = \frac{V}{\lambda_1} = \frac{V}{4l}$$

Fundamental frequency, first Harmonic motion in closed organ pipe.

Second mode of vibrations

$$\lambda_2 = \frac{4l}{2\times2-1} = \frac{4l}{3}, \quad \text{when } n=2$$

$$l = \frac{3\lambda_2}{4}$$

$$\upsilon_2 = \frac{V}{\lambda_2} = \frac{V}{\frac{4l}{3}} = \frac{3V}{4l}$$

$$\upsilon_2 = 3\upsilon_1$$

Thrid Harmonic motion in closed organ pipe.

$$\lambda_3 = \frac{4l}{2\times3-1} = \frac{4l}{5}, \quad \upsilon_3 = \frac{V}{\lambda_3} = \frac{V}{\frac{4l}{5}} = \frac{5V}{4l}$$

$$l = \frac{5\lambda_3}{4}$$

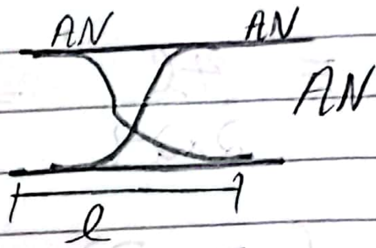
$$\lambda = \frac{4l}{2n-1}$$

Fifth Harmonic motion $= 5\upsilon_1$

$\nu_1, \nu_3, \nu_5 \ldots\ldots$ are harmonics that are present in standing waves in closed organ pipe.
$\therefore$ Odd harmonics are present.

V. <u>Standing waves in a open organ pipe.</u>

First mode of ~~Harmo~~ Vibrations



$$l = \frac{\lambda_1}{2}$$

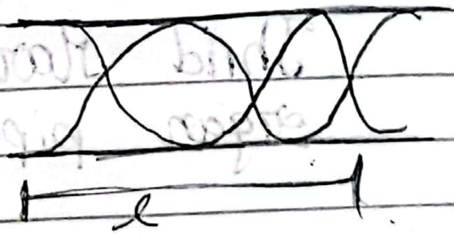
$$\lambda_1 = 2l$$

$$\nu_1 = \frac{V}{\lambda_1} = \frac{V}{2l}$$

First Harmonic motion

Second mode of Vibration

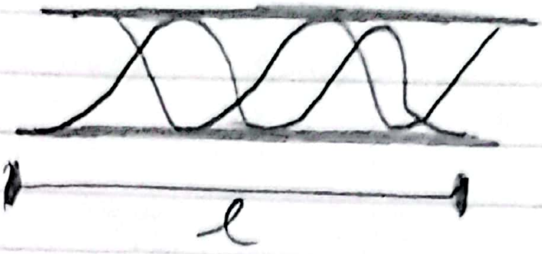
$$l = \lambda_2 \left(\frac{\lambda}{2} + \frac{\lambda}{2}\right)$$



$$\nu_2 = \frac{V}{\lambda_2} = \frac{V}{l} = 2\nu_1$$

Second Harmonic Motion.

Third Mode of Vibration



$$l = \frac{3\lambda_3}{2}$$

$$3\lambda_3 = 2l$$

$$\lambda_3 = \frac{2l}{3}$$

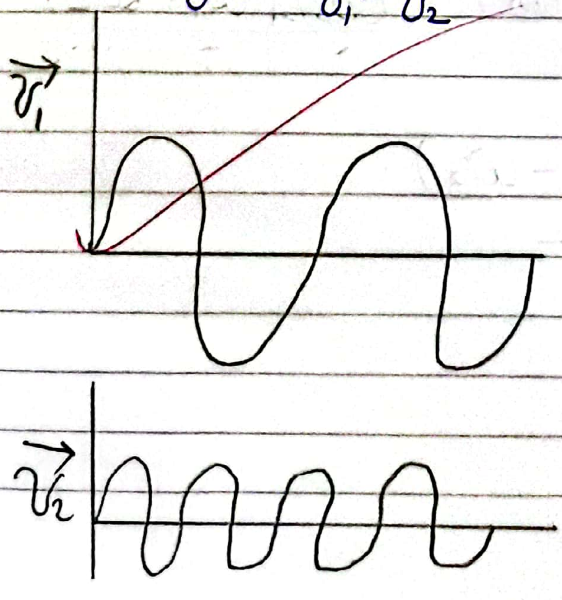
$$\nu_3 = \frac{V}{\lambda_3} = \frac{V}{\frac{2l}{3}} = \frac{3V}{2l}$$

$$V_3 = 3\nu_1$$

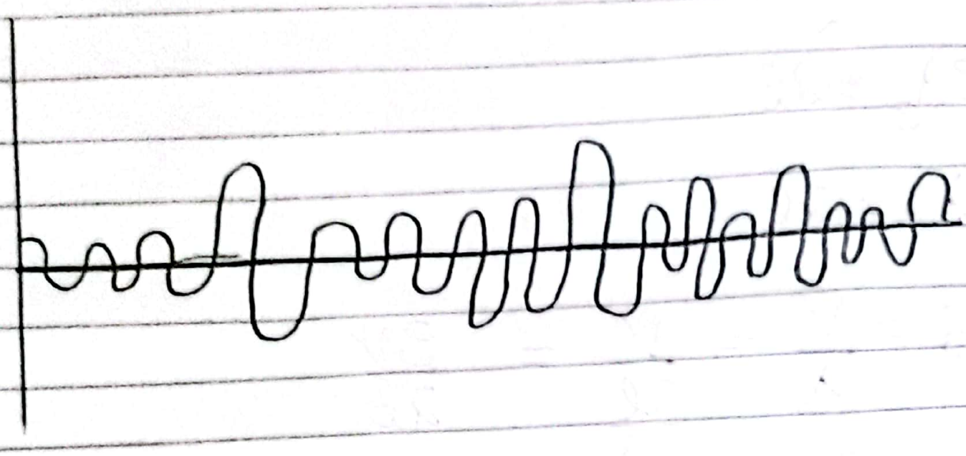
In an organ open pipe, contineus Harmonic are present.

c. **Beats**: The phenomena of alternate rise or fall in the intensity of sound with time when two waves of nearly same frequencies travelling along the same line and in the same direction, superimpose on each other, it's known as beats.

$$\nu = \nu_1 - \nu_2$$



Net Displacement



i. Derive Beat frequency

$$y_1 = A\sin(\omega_1 t - k_1 x)$$
$$y_2 = A\sin(\omega_2 t - k_2 x)$$
$$\vec{y} = \vec{y_1} + \vec{y_2}$$
$$= A\sin\omega_1 t + A\sin\omega_2 t$$

[$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$]

$$= A(\sin\omega_1 t + \sin\omega_2 t)$$

$$= 2A\sin\frac{\omega_1 t + \omega_2 t}{2}\ \cos t\frac{(\omega_1 - \omega_2)}{2}$$

$$\text{Amplitude} = \cos t\frac{(\omega_1 - \omega_2)}{2}$$

For maximum intensity,

$\cancel{\cos\theta}$ $\cos t = 0, \pi, 2\pi, 3\pi, \ldots, n\pi = 1$

$$\frac{t(\omega_1 - \omega_2)}{2} \neq 0$$

$$\cancel{t\left(\frac{2\pi v_1 - 2\pi v_2}{2}\right) = 0} \quad \left[\omega = \frac{2\pi}{T} = 2\pi v\right]$$

$$\cancel{t \times 2\pi\left(\frac{v_1 - v_2}{2}\right) = 0}$$

$$\cancel{t = 0}$$

$$\frac{t\,2\pi(v_1 - v_2)}{2} = \pi$$

$$t = \frac{1}{v_1 - v_2} \qquad v = \frac{1}{v_1 - v_2} \quad \text{— (1)}$$

$$v = \frac{1}{t} = v_1 - v_2$$

$$v = v_1 - v_2$$

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