Iyad Imtiyaz
X B 51965

# 2 Polynomials

In polynomial power of variable must be a positive integer. For example $7x^2+3x+5$.

A linear polynomial has atmost 1 zero (1 or less than 1). For example $7x+5$

A quadratic polynomial has at most 2 zeroes [2 or less than 2]. For example $5x^2+8x+3$

A cubic polynomial has atmost 3 zeroes (3 or less than 3). For example $x^3+x^2+x+10$

Standard form of a Quaratic Polynomial

Sum of zeroes = $-b/a$
Product of ~~fof~~ zeroes = $c/a$ = constant term/coeff of $x^2$

Exercise 2.1

1) The graphs of $y=p(x)$ are given in textbook, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

i) Graph of polynomial does not meet $x$ axis.

ii) Graph of polynomial cuts $x$ axis only once. So the polynomial has exactly one zero.

iii) Graph of polynomial cut $x$ axis thrice. So the polynomical has 3 zeros.

iv) Graph of polynomial cut $x$ axis twice. So the polynomical has exactly 2 ~~re~~zeroes.

Exercise 2.2

1) Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

a) $x^2 - 2x - 8$

Splitting the middle term

product = $-8$
sum = $-2$
$-4 \times 2 = -8$
$-4 + 2 = -2$

$x^2 - 4x + 2x - 8$

$x(x-4) + 2(x-4)$

$(x+2)(x-4)$

$x + 2 = 0$

$x = -2$
$\alpha = -2$

$x - 4 = 0$
$x = 4$
$\beta = 4$

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Verification

$ax^2 + bx + c$

$x^2 - 2x - 8$

$a = 1, b = -2, c = -8$

$a = 1 \ , b = -2 \ , c = -8$

$\alpha + \beta = \frac{-b}{a}$

$LHS = \alpha + \beta$

$= -2 + 4 = 2$

$RHS = \frac{-b}{a}$

$= \frac{-(-2)}{1} = 2$

$\alpha\beta = \frac{c}{a}$

$LHS = \alpha\beta$

$= (-2)(4) = -8$

$RHS = \frac{c}{a}$

$= \frac{-8}{1} = -8$

ii) $4u^2 + 8u$

$4u(u+2)$

$4u = 0, \quad u+2 = 0$

$u = 0, \quad \beta = -2$
$\alpha = 0, \quad \beta = -2$

Verification

$ax^2 + bx + c$
$4u^2 + 8u + 0$

$a = 4, \ b = 8, \ c = 0$

$\alpha + \beta = -b/a$

$LHS = 0 + (-2) = -2$

$RHS = -b/a = -8/4 = -2$

$\alpha\beta = \frac{c}{a}$

$LHS = (0)(-2) = 0$

$RHS = \frac{c}{a} = \frac{0}{4} = 0$

iv) $t^2 - 15$

$(t)^2 - (\sqrt{15})^2 = (t + \sqrt{15})(t - \sqrt{15})$

$t + \sqrt{15} = 0, \quad t - \sqrt{15} = 0$

$t = -\sqrt{15}, \quad t = \sqrt{15}$

$\alpha = -\sqrt{15}, \quad \beta = \sqrt{15}$

Verification

$ax^2 + bx + c$

$t^2 + 0(t) - 15$

$a = 1, b = 0, \quad c = -15$

$\alpha + \beta = \frac{-b}{a}$

$LHS = \alpha + \beta$

$= -\sqrt{15} + \sqrt{15} = 0$

$RHS = \frac{-0}{1} = 0$

$\alpha\beta = \frac{c}{a}$

$LHS = \alpha\beta = -\sqrt{15} \times \sqrt{15} = -15$

$RHS = \frac{-15}{1} = -15$

(iv) $4u^2 + 8u$

$4u(u+2)$

$4u = 0 \Rightarrow u = 0 = \alpha$

$u + 2 = 0 \Rightarrow u = -2 = \beta$

Verification

$ax^2 + bx + c$

$a = 4, \quad b = 8, \quad c = 0$

$\alpha + \beta = \frac{-b}{a}$

$LHS = 0 + (-2) = -2$

$RHS = \frac{-(+8)}{4} = -2$

$\alpha\beta = \frac{c}{a}$

$LHS = 0 \times -2 = 0$

$RHS = \frac{c}{a} = \frac{0}{4} = 0$

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iii) $6x^2 - 3 - 7x$

$6x^2 - 7x - 3$

$6x^2 - 9x + 2x - 3$

$3x(2x - 3) + 1(2x - 3)$

$(3x + 1)(2x - 3)$

$3x + 1 = 0 \Rightarrow 3x = -1 \Rightarrow x = \frac{-1}{3} = \alpha$

$2x - 3 = 0 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2} = \beta$

Verification

$ax^2 + bx + c$

$a = 6$
$b = -7$
$c = -3$

$\alpha + \beta = \frac{-b}{a}$

$RHS = \frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$

$LHS = \alpha + \beta = \frac{-1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6}$

$\alpha\beta = \frac{c}{a}$

$LHS = \alpha\beta = \frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6} = -\frac{1}{2}$

$RHS = \frac{-3}{6} = -\frac{1}{2}$

iv) $4u^2 + 8u$

$4u(u+2)$

$4u = 0 \Rightarrow u = \frac{0}{4} = 0 = \alpha$

$u+2 = 0 \Rightarrow u = -2 = \beta$

Verification

$ax^2 + bx + c$

$a = 4, \quad b = 8, \quad c = 0$

$\alpha + \beta = \frac{-b}{a}$

$LHS = \alpha + \beta = 0 + (-2) = -2$

$RHS = \frac{-b}{a} = \frac{-8}{4} = -2$

$\alpha\beta = \frac{c}{a}$

$LHS = \alpha\beta = 0 \times -2 = 0$

$RHS = c/a = 0/4 = 0$

vi) $3x^2 - x - 4$

$3x^2 - 4x + 3x - 4$

$3x^2 + 3x - 4x - 4$

$3x(x+1) - 4(x+1)$

$(3x-4)(x+1)$

$3x - 4 = 0 \Rightarrow 3x = 4 \Rightarrow x = \frac{4}{3} = \alpha$
$x + 1 = 0 \Rightarrow x = -1 = \beta$

Verification

$ax^2 + bx + c$

$a = 3$
$b = -1$
$c = -4$

$\alpha + \beta = \frac{-b}{a}$

$LHS = \frac{4}{3} + \frac{-1}{1} = \frac{4-3}{3} = \frac{1}{3}$

$RHS = \frac{-(-1)}{3} = \frac{1}{3}$

$\alpha\beta = \frac{c}{a}$

$LHS = \frac{4}{3} \times -1 = \frac{-4}{3}$

$RHS = c/a = -4/3$

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2) Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

a) $\frac{1}{4}$, $-1$

$S = \frac{1}{4}$

$P = -1$

$p(x) = x^2 - (S)x + P$

$= x^2 - \left(\frac{1}{4}\right)x + (-1)$

$= \frac{x^2}{1} - \frac{x}{4} - \frac{1}{1}$

$= \frac{4x^2 - x - 4}{4}$

$= \frac{1}{4}(4x^2 - x - 4)$.

2nd Method

Sum of zeroes $= \frac{-b}{a} = \frac{1}{4}$

Product of zeroes $= \frac{c}{a} = \frac{-1}{1}$

$\frac{c}{a} = \frac{-1}{1} \times \frac{4}{4} = \frac{-4}{4}$

$a = 4,\ b - 1,\ c = -4$

$ax^2 + bx + c$

$4x^2 - x - 4$

Quadractic equation :- $4x^2 - x - 4$

ii) $\sqrt{2}$, $\frac{1}{3}$

$S = \sqrt{2}$

$P = \frac{1}{3}$

$p(x) = x^2 - (S)x + P$

$= \frac{x^2}{1} - \frac{(\sqrt{2})x}{1} + \frac{1}{3} \Rightarrow \frac{3x^2 - (3\sqrt{2})x + 1}{3}$

$= \frac{1}{3}(3x^2 - 3\sqrt{2}x + 1)$

Quadric equation :- $3x^2 - 3\sqrt{2}x + 1$

iii) $0, \sqrt{5}$

$S = 0$
$P = \sqrt{5}$

$p(x) = x^2 - (S)x + P$

$= x^2 - (0)x + \sqrt{5}$

$= x^2 + \sqrt{5}$

2nd Method

Sum of zeros $= \frac{-b}{a} = \frac{0}{1}$

Product of zeros $= \frac{c}{a} = \frac{\sqrt{5}}{1}$

$a = 1, b = 0, c = \sqrt{5}$

$ax^2 + bx + c$

$x^2 + 0 \cdot x + \sqrt{5}$

$x^2 + \sqrt{5}$.

Quadratic equation :- $x^2 + \sqrt{5}$

iv) $1, 1$

$S = 1$
$P = 1$

$p(x) = x^2 - (S)x + P$

$= x^2 - (1)x + 1$

$= x^2 - x + 1$

2nd meathod

Sum of zeros $= \frac{-b}{a} = \frac{1}{1} =$

$b = -1, a = 1$

Product of zeros $= \frac{c}{a} = \frac{1}{1}$

$c = 1$

$ax^2 + bx + c$

$x^2 - x + 1$

Quadratic equation $= x^2 - x + 1$

Method - 1

v) $\frac{-1}{4}, \frac{1}{4}$

$S = \frac{-1}{4}$

$P = \frac{1}{4}$

$p(x) = x^2 - (S)x + P$

$= x^2 - (-1/4)x + 1/4$

$= \frac{x^2}{1} + \frac{x}{4} + \frac{1}{4}$

$= \frac{4x^2 + x + 1}{4}$

$= \frac{1}{4}(4x^2 + x + 1)$

Method - 2

Sum of zero $= \frac{-b}{a} = +\left(\frac{-1}{4}\right)$

$\frac{-b}{a} = -1/4$

$b/a = 1/4.$

product of zeros $= \frac{c}{a} = 1/4$

$ax^2 + bx + c.$

$a = 4, b = 1, c = 1$

$4x^2 + x + 1$

Quadratic equation $= 4x^2 + x + 1$

vi) 4, 1

$S = 4$

$P = 1$

$p(x) = x^2 - (S)x + P$

$= x^2 - (4)x + 1 = x^2 - 4x + 1$

Quadratic equation $= x^2 - 4x + 1$

Exercise 2.3

1) Divide

a) $x^3 - 3x^2 + 5x - 3 \div x^2 - 2$

$$x - 3$$
$$x^2 - 2 \,\overline{)\, x^3 - 3x^2 + 5x - 3}$$
$$x^3 \qquad\qquad -2x$$
$$(-) \qquad\qquad (+)$$
$$\overline{\qquad 7x - 9}$$

$Q = x - 3$
$R = 7x - 9$

b) $x^4 - 3x^2 + 4x + 5 \div x^2 + 1 - x$

$$x^2 + x - 3$$
$$x^2 + 1 - x \,\overline{)\, x^4 - 3x^2 + 4x + 5}$$
$$x^4 + x^2 \qquad\qquad -x^3$$
$$(-) \; (-) \qquad\qquad (+)$$
$$\overline{-4x^2 + 4x + 5 + x^3}$$
$$-x^2 \quad + x \quad + x^3$$
$$(+) \quad (-) \quad (-)$$
$$\overline{-3x^2 + 3x + 5}$$
$$-3x^2 + 3x - 3$$
$$(+) \quad (-) \; (+)$$
$$\overline{\qquad 8}$$

$Q = x^2 + x - 3$
$R = 8$

c) $x^4-5x+6 \div 2-x^2$

$$
\begin{array}{r}
-x^2-2 \\
-x^2+2 \,\big)\, x^4-5x+6 \\
x^4 \; \cancel{-5x} \quad -2x^2 \\
(-) \qquad\qquad (+) \\
\hline
-5x+6+2x^2 \\
-4+2x^2 \\
\hline
-5x+10
\end{array}
$$

$Q = -x-2$
$R = -5x+10$

2) Divide

a) $t^2-3$ , $2t^4+3t^3-2t^2-9t-12$

$$
\begin{array}{r}
2t^2+3t-4 \\
t^3-3 \,\big)\, 2t^4+3t^3-2t^2-9t-12 \\
\cancel{2t^4} \qquad 6t^2 \\
(-) \qquad (+) \\
\hline
\cancel{3t^3}-4t^2-\cancel{9t}-12 \\
\cancel{3t^3} \qquad \cancel{9t} \\
(-) \qquad (+) \\
\hline
-4t^2 \qquad -12 \\
-4t^2 \qquad -12 \\
(+) \qquad (+) \\
\hline
0
\end{array}
$$

$Q = 2t^2+3t-4$
$R = 0$
Yes it is a Factor

b) $x^2+3x+1$ , $3x^4+5x^3-7x^2+2x+2$

$$3x^2-4x+2$$
$$x^2+3x+1\,\overline{)\,3\cancel{x^4}+5x^3-7x^2+2x+2}$$
$$3\cancel{x^4} \quad 9x \quad 3x^2$$
$$(-) \quad (-) \quad (-)$$
$$-4\cancel{x^3}-10x^2+2x+2$$
$$-\cancel{4}x^3-12x^2-4x$$
$$(+) \quad (+) \quad (+)$$
$$2x^2+6x+2$$
$$2x^2+6x+2$$
$$(-) \quad (-) \quad (-)$$
$$0$$

$Q=3x^2-4x+2$
$R=0$

Yes, its a factor

c) $x^3-3x+1$ , $x^5-4x^3+x^2+3x+1$

$$x^2-1$$
$$x^3-3x+1\,\overline{)\,\cancel{x^5}-4x^3+\cancel{x^2}+3x+1}$$
$$\cancel{x^5}-3x^3 \quad \cancel{x^2}$$
$$(-) \quad (+) \quad (-)$$
$$-x^3 \quad +3x+1$$
$$-x^3 \quad 3x-1$$
$$(+) \quad (-) \quad (+)$$
$$2$$

$Q=x^2-1$
$R=2$

No, it isn't a factor

3) Obtain all other zeros of $3x^4+6x^3-2x^2-10x-5$, if two of its zeros are $\sqrt{5/3}$ and $-\sqrt{5/3}$.

a) ~~let let~~ $\cancel{3x^4}$

let $\alpha = \sqrt{5/3}$ , $\beta = -\sqrt{5/3}$

Sum $= \alpha+\beta = \sqrt{5/3} + (-\sqrt{5/3}) = 0$

Product $= \alpha\beta = (\sqrt{5/3})(-\sqrt{5/3}) = -5/3$

$p(x) = x^2 - (s)x + p$

$= x^2 - (0)x + (-5/3)$

$= x^2 - 5/3$ (LCM = 3)

$= \dfrac{3x^2-5}{3}$

$= \frac{1}{3}(3x^2-5)$

$p(x) = 3x^2-5$

$$x^2+2x+1$$

$$3x^2-5 \,\big)\, 3x^4+6x^3-2x^2-10x-5$$

$$\cancel{3x^4} \quad -5x^2$$

$$(-) \quad (+)$$

$$6x^3+3x^2-10x-5$$

$$\cancel{6x^3} \quad -10x$$

$$(-) \quad (+)$$

$$3x^2-5$$

$$\cancel{3x^2} \quad \cancel{-5}$$

$$(-) \quad (+)$$

$$0$$

$Q = x^2+2x+1$

$R = 0$

Zero ~~fo~~ of polynomial $p(x) = x^2+2x+1$

$$= x^2 + x + x + 1$$

$$x(x+1)+1(x+1)$$

$$(x+1)(x+1)$$

Zero of polynomial $= -1$

There the zeros of the polynomial of zeros are

$-1, \sqrt{5/3}, -\sqrt{5/3}, 3x^2-5$

4) On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

$p(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$

$r(x) = -2x + 4$

$g(x) = ?$

divident = dr × q + r

$p(x) = g(x) \times q(x) + r(x)$

$$g(x) = \frac{p(x) - r(x)}{q(x)}$$

$$= \frac{x^3 - 3x^2 + x + 2 - [-2x + 4]}{(x - 2)}$$

$$= \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}$$

$$= \frac{x^3 - 3x^2 + x + 2x + 2 - 4}{x - 2}$$

$$= \cancel{x^3 - 3x^2 + x + 2x + 2}$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$x^2 \cancel{+} - x + 1$$

$$x - 2 \,\overline{\big)\, \cancel{x^3} - 3x^2 + x + 2x + 2 - 4}$$

$$\cancel{x^3} - 2x^2$$

$$(-) \quad (+)$$

$$\cancel{x^2} + x + 2\cancel{x} + 2 - 4$$

$$(-)\,\cancel{x^2} \quad (+) - 2x$$

$$x - 2$$

$$x - 2$$

$$0$$

$Q = x^2 + x + 1$

~~x-2~~

$g(x) = x^2 \cancel{+} - x + 1$